Université PANTHÉON-ASSAS (PARIS II) U.E.C. 2/4211
Droit-Economie-Sciences Sociales Assas

Session :	Septembre 2022.
Année d'étude :	M1
Discipline :	***Optimization : Theory and Algorithms*** (Unité d'Enseignements Complémentaire 2).
Titulaire du cours :	M. Lorenzo BASTIANELLO.
Durée de l'épreuve :	1h30.
Document(s) autorisé(s) :	Calculatrice autorisée. Le téléphone portable n'est pas autorisé comme calculette. Documents interdits, ainsi que tout appareil électronique permettant une connexion à distance.

Exam of Optimization : Theory and Algorithms (4211)

Instructions :

— A bad and messy presentation may result in negative points.
— Vectors are indicated in bold letters.
— *The score of each exercise is given only as a benchmark and it may change.*

Exercise 1. (5 points) Suppose that a differentiable function $f : \mathbb{R} \to \mathbb{R}$ is unimodal over an interval $[a, b]$. Denote $x^* \in (a, b)$ its unique local minimum.

1. (1 point) Let $x \in (a, b)$ and suppose that $f'(x) > 0$. Find an interval I strictly contained in $[a, b]$ such that $x^* \in I$.
2. (3 points) Describe the bisection algorithm to find x^*.
3. (1 point) Show that the bisection algormithm converges linearly with convergence ratio $\frac{1}{2}$.

Exercise 2. (5 points) Consider the function $f(x, y) = (x + y^2)^2$. At the point $\boldsymbol{x_0} = (1, 0)$ we consider the search direction $\boldsymbol{p} = (-1, 1)$.

1. (1 point) Give the definition of directional derivative.
2. (3 points) Give the definition of descent direction and show that $\boldsymbol{p}$ is a descent direction.
3. (1 point) Find all $\boldsymbol{x}^*$ that minimize f.

Exercise 3. (3 points) We study speed of convergence.

1. (1 point) Give the definition of superlinear and quadratic convergence.
2. (2 points) Show that the sequence $x_k = 1 + \left(\frac{1}{2}\right)^{2^k}$ converges quadratically to 1.

Exercise 4. (7 points) Consider the function $f(x, y) = 100(y - x^2)^2 + (1 - x)^2$.

1. (2 points) Compute ∇f, $\nabla^2 f$.
2. (2 points) Find the unique global minimizer $\boldsymbol{x}^*$ of f. Justify why it is a global minimizer. Evaluate the gradient and Hessian at $\boldsymbol{x}^*$.
3. (3 points) Write down a steepest descent direction algorithm to minimize f. At each step, use the backtracking algorithm to find the step length α.