Séries Temporelles

Master I Economie-Gestion
Mention Ingénierie Economique et Statistique
Mention Monnaie-Finance-Banque

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Session Septembre 2018

1. **Exercice 1** (4 pts.)
Soit le modèle ARMA(p,q) stationnaire et inversible associé à la série X_t :

$$\phi(B) X_t = c + \theta(B) \varepsilon_t \ , \ t = 1, ..., T$$

où $\phi(B) = (1 - \phi_1 B - ... - \phi_p B^p)$ et $\theta(B) = (1 - \theta_1 B - ... - \theta_q B^q)$ sont des polynômes d'opérateur de retard, ε_t est un bruit blanc $\left(\varepsilon_t \sim iid\left(0, \sigma_\varepsilon^2\right)\right)$ et c, une constante.

   (a) Soit
   $$\widehat{X}_T(\tau) = E[X_{T+\tau} | I_T]$$
   la valeur prévue de X à l'horizon $(T + \tau)$, où I_T est l'ensemble informationnel disponible à la date T. Exprimez $\widehat{X}_T(\tau)$ comme une combinaison linéaire infinie des perturbations présente et passées.

   (b) Déterminez l'expression de l'intervalle de prévision à 95% de sécurité.

2. **Exercice 2** (6 pts.)
On considère le cours de clôture X_t de l'action Facebook au NYSE du 18 mai 2012 au 21 novembre 2013. Les rendements composés (log returns) sont donnés par
$x_t = \Delta \log(X_t) = \log(X_t) - \log(X_{t-1})$.

   (a) Commentez les profils d'évolution de l'action Facebook et de ses rendements composés (voir figures 1 et 2).

   (b) On estime un modèle EGARCH(1,1) sur les rendements du type:

$$\left\{ \begin{array}{rcc} x_t & = & \beta_0 + a_t, \ a_t = \sigma_t \varepsilon_t \ , \ \varepsilon_t \sim N(0,1) \\ \log\left(\sigma_t^2\right) & = & \frac{\alpha_0 + g(\varepsilon_{t-1})}{1 - \beta B} \\ g(\varepsilon_{t-1}) & = & \alpha_1 \varepsilon_{t-1} + \alpha_2 |\varepsilon_{t-1}| \end{array} \right.$$

   Commentez les résultats de l'estimation (table 1).

   (c) A l'aide des valeurs estimées des paramètres, exprimez la volatilité σ_t^2 (et non le log de la volatilité) sous forme d'un modèle GARCH à seuil.

   (d) Pour un choc de 2 écarts-types, montrez que l'impact d'un choc négatif est plus important que l'impact d'un choc positif de même amplitude.

3. **Exercice 3** (10 points) - On s'intéresse aux logarithmes des taux d'intérêt des bons du Trésor américain à maturité d'un an ($\log\_1y$) (*Treasury bills*) et à maturité de 3 ans ($\log\_3y$) (*Treasury notes*) sur la période avril 1953 - janvier 2001 (figure 3).

   (a) On effectue d'abord des tests de racine unitaire sur la série $\log\_1y$. Commentez les résultats des tables 2, 3 et 4. Que concluez-vous?

   (b) On suppose que la série $\log\_3y$ est $I(1)$ et on définit les variables en différences premières: $d\log\_1y$ et $d\log\_3y$. Commentez la table 5 sur la sélection du nombre de retards pour un VAR à deux variables ($d\log\_1y$, $d\log\_3y$). Commentez la table 6 du test de causalité "à la Granger".

(c) On souhaite estimer une relation de long terme entre les logarithmes des taux d'intérêt:

$$\log\_1y_t = \beta_0 + \beta_1 \log\_3y_t + \beta_2 dummy_t + \varepsilon_t$$

où $dummy_t$ est une variable indicatrice valant 1 pour la période 1980.1 - 1985.12 et 0 sinon. Commentez les tables 7 et 8. La relation est-elle cointégrée? Justifiez votre réponse.

(d) On estime un processus VAR non cointégré. Commentez les tables 9 à 11 et la figure 4.

Aucun document autorisé.

Calculatrices et tables statistiques autorisées.

Table des valeurs critiques du test de Dickey-Fuller pour $\rho = 1$

T	1%	5%	10%
	modèle (1)		
100	−2.60	−1.95	−1.61
250	−2.58	−1.95	−1.62
500	−2.58	−1.95	−1.62
∞	−2.58	−1.95	−1.62
	modèle (2)		
100	−3.51	−2.89	−2.58
250	−3.46	−2.88	−2.57
500	−3.44	−2.87	−2.57
∞	−3.43	−2.86	−2.57
	modèle (3)		
100	−4.04	−3.45	−3.15
250	−3.99	−3.43	−3.13
500	−3.98	−3.42	−3.13
∞	−3.96	−3.41	−3.12

Table des valeurs critiques de la constante et de la tendance, tests de Dickey-Fuller

	Modèle (2)			Modèle (3)					
	Constante			Constante			Tendance		
T	1%	5%	10%	1%	5%	10%	1%	5%	10%
100	3.22	2.54	2.17	3.78	3.11	2.73	3.53	2.79	2.38
250	3.19	2.53	2.16	3.74	3.09	2.73	3.49	2.79	2.38
500	3.18	2.52	2.16	3.72	3.08	2.72	3.48	2.78	2.38
∞	3.18	2.52	2.16	3.71	3.08	2.72	3.46	2.78	2.38

Table des valeurs critiques des tests DF de cointégration avec constante

variables			
(y compris Y_t)	1%	5%	10%
2	−3.90	−3.34	−3.04
3	−4.29	−3.74	−3.45
4	−4.64	−4.10	−3.81
5	−4.96	−4.42	−4.13

Table des valeurs critiques du test CRDW à 5%

variables			
(y compris Y_t)	$T = 50$	$T = 100$	$T = 200$
2	0.72	0.38	0.20
3	0.89	0.48	0.25
4	1.05	0.58	0.30
5	1.19	0.68	0.35

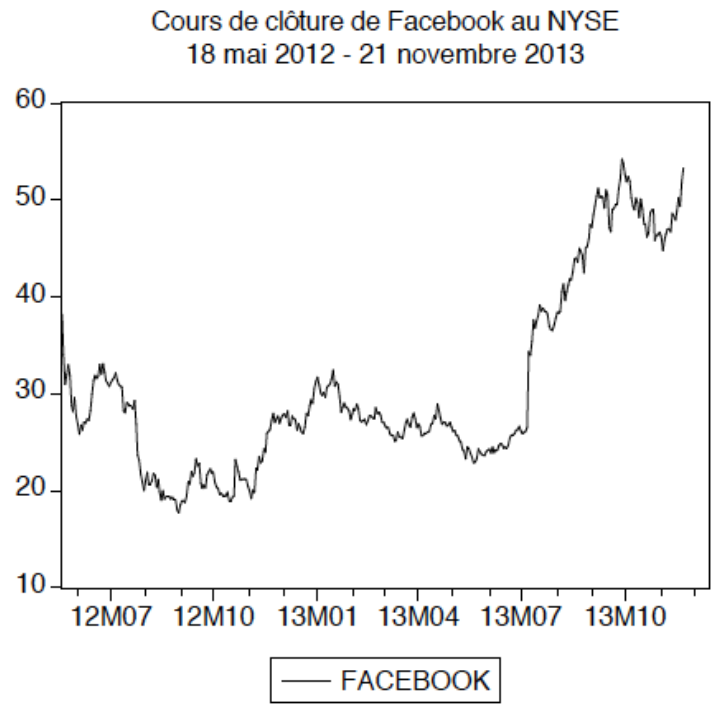

Figure 1: Cours de clôture de l'action Facebook au NYSE du 18 mai 2012 au 21 novembre 2013.

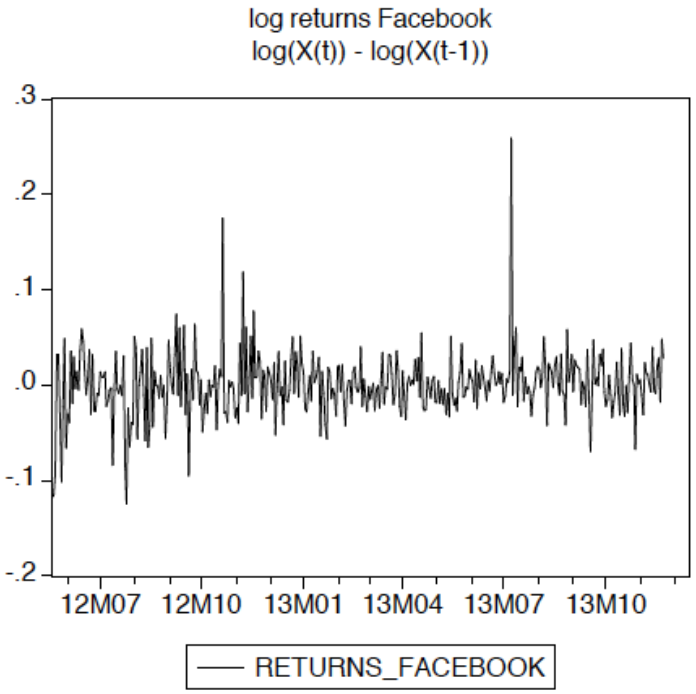

Figure 2: Rendements composés (log returns) de l'action Facebook au NYSE du 18 mai 2012 au 21 novembre 2013.

Dependent Variable: RETURNS_FACEBOOK
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/14/13 Time: 10:22
Sample (adjusted): 5/21/2012 11/21/2013
Included observations: 394 after adjustments
Convergence achieved after 10 iterations
Variance backcast: ON
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001600	0.001612	0.992574	0.3209
Variance Equation				
C(2)	-0.298726	0.000213	-1401.717	0.0000
C(3)	-0.019668	0.004712	-4.174110	0.0000
C(4)	-0.031841	0.014960	-2.128446	0.0333
C(5)	0.955320	0.000437	2186.111	0.0000

R-squared	-0.000509	Mean dependent var	0.000844
Adjusted R-squared	-0.010797	S.D. dependent var	0.033541
S.E. of regression	0.033722	Akaike info criterion	-3.986524
Sum squared resid	0.442360	Schwarz criterion	-3.936063
Log likelihood	790.3453	Durbin-Watson stat	1.908211

Table 1: Estimation EGARCH des rendements composés (log returns) de l'action Facebook.

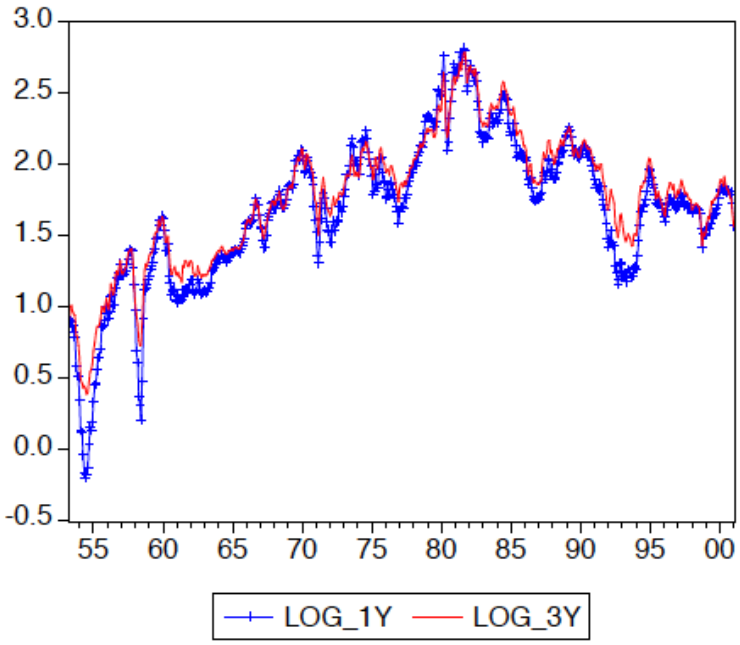

Figure 3: Logarithmes des taux d'intérêt des bons du Trésor américain à maturité d'un an (log _1y) (*Treasury bills*) et à maturité de 3 ans (log _3y) (*Treasury notes*) sur la période avril 1953 - janvier 2001.

Augmented Dickey-Fuller Unit Root Test on LOG_1Y

Null Hypothesis: LOG_1Y has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 6 (Automatic based on AIC, MAXLAG=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.479609	0.3382
Test critical values:	1% level	-3.974323	
	5% level	-3.417765	
	10% level	-3.131321	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG_1Y)
Method: Least Squares
Date: 12/07/12 Time: 17:48
Sample (adjusted): 1953M11 2001M01
Included observations: 567 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG_1Y(-1)	-0.014785	0.005962	-2.479609	0.0134
D(LOG_1Y(-1))	0.482140	0.041563	11.60023	0.0000
D(LOG_1Y(-2))	-0.135127	0.046254	-2.921408	0.0036
D(LOG_1Y(-3))	0.073103	0.046610	1.568413	0.1174
D(LOG_1Y(-4))	0.000719	0.046656	0.015405	0.9877
D(LOG_1Y(-5))	0.031125	0.046250	0.672972	0.5012
D(LOG_1Y(-6))	-0.102710	0.041863	-2.453445	0.0145
C	0.023793	0.008728	2.726195	0.0066
@TREND(1953M04)	7.95E-06	1.85E-05	0.428819	0.6682

R-squared	0.220265	Mean dependent var	0.001743
Adjusted R-squared	0.209086	S.D. dependent var	0.067589
S.E. of regression	0.060109	Akaike info criterion	-2.769556
Sum squared resid	2.016130	Schwarz criterion	-2.700662
Log likelihood	794.1692	F-statistic	19.70341
Durbin-Watson stat	1.988135	Prob(F-statistic)	0.000000

Table 2: Test de racine unitaire de la série log _1y.

Augmented Dickey-Fuller Unit Root Test on LOG_1Y

Null Hypothesis: LOG_1Y has a unit root
Exogenous: Constant
Lag Length: 6 (Automatic based on AIC, MAXLAG=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.692981	0.0758
Test critical values:	1% level	-3.441654	
	5% level	-2.866419	
	10% level	-2.569428	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG_1Y)
Method: Least Squares
Date: 12/07/12 Time: 17:48
Sample (adjusted): 1953M11 2001M01
Included observations: 567 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG_1Y(-1)	-0.013371	0.004965	-2.692981	0.0073
D(LOG_1Y(-1))	0.481417	0.041498	11.60085	0.0000
D(LOG_1Y(-2))	-0.136027	0.046173	-2.946023	0.0034
D(LOG_1Y(-3))	0.072228	0.046531	1.552261	0.1212
D(LOG_1Y(-4))	-0.000242	0.046568	-0.005191	0.9959
D(LOG_1Y(-5))	0.030377	0.046183	0.657744	0.5110
D(LOG_1Y(-6))	-0.104191	0.041690	-2.499178	0.0127
C	0.023727	0.008720	2.720977	0.0067

R-squared	0.220008	Mean dependent var	0.001743
Adjusted R-squared	0.210240	S.D. dependent var	0.067589
S.E. of regression	0.060065	Akaike info criterion	-2.772754
Sum squared resid	2.016794	Schwarz criterion	-2.711515
Log likelihood	794.0758	F-statistic	22.52480
Durbin-Watson stat	1.988824	Prob(F-statistic)	0.000000

Table 3: Test de racine unitaire de la série log _1y.

Augmented Dickey-Fuller Unit Root Test on LOG_1Y

Null Hypothesis: LOG_1Y has a unit root
Exogenous: None
Lag Length: 6 (Automatic based on AIC, MAXLAG=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.304156	0.5760
Test critical values:	1% level	-2.569047	
	5% level	-1.941383	
	10% level	-1.616324	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG_1Y)
Method: Least Squares
Date: 12/07/12 Time: 17:50
Sample (adjusted): 1953M11 2001M01
Included observations: 567 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG_1Y(-1)	-0.000440	0.001446	-0.304156	0.7611
D(LOG_1Y(-1))	0.481035	0.041735	11.52598	0.0000
D(LOG_1Y(-2))	-0.142215	0.046380	-3.066304	0.0023
D(LOG_1Y(-3))	0.067465	0.046763	1.442697	0.1497
D(LOG_1Y(-4))	-0.006237	0.046781	-0.133324	0.8940
D(LOG_1Y(-5))	0.025234	0.046408	0.543734	0.5868
D(LOG_1Y(-6))	-0.112672	0.041810	-2.694817	0.0073

R-squared	0.209677	Mean dependent var	0.001743
Adjusted R-squared	0.201209	S.D. dependent var	0.067589
S.E. of regression	0.060408	Akaike info criterion	-2.763124
Sum squared resid	2.043506	Schwarz criterion	-2.709539
Log likelihood	790.3456	Durbin-Watson stat	1.987490

Table 4: Test de racine unitaire de la série $\log\_1y$.

VAR Lag Order Selection Criteria
Endogenous variables: DLOG_1Y DLOG_3Y
Exogenous variables: C
Date: 12/07/12 Time: 17:04
Sample: 1953M04 2001M01
Included observations: 568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	2084.911	NA	2.24e-06	-7.334193	-7.318904	-7.328227
1	2148.215	125.9393	1.82e-06	-7.543010	-7.497143	-7.525111
2	2163.568	30.43682*	1.75e-06	-7.582988	-7.506542*	-7.553156*
3	2167.712	8.184864	1.74e-06*	-7.583493*	-7.476469	-7.541729
4	2168.944	2.424325	1.76e-06	-7.573745	-7.436143	-7.520049
5	2173.051	8.055298	1.76e-06	-7.574123	-7.405942	-7.508493

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 5: Sélection du nombre de retards pour le processus VAR à deux variables ($d\log\_1y$, $d\log\_3y$).

Pairwise Granger Causality Tests
Date: 12/08/12 Time: 10:28
Sample: 1953M04 2001M01
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Probability
DLOG_3Y does not Granger Cause DLOG_1Y	571	4.24235	0.01483
DLOG_1Y does not Granger Cause DLOG_3Y		2.24592	0.10677

Table 6: Tests de causalité "à la Granger" pour les variables $d\log\_1y$ et $d\log\_3y$

Dependent Variable: LOG_1Y
Method: Least Squares
Date: 12/07/12 Time: 16:56
Sample: 1953M04 2001M01
Included observations: 574

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.361031	0.016738	-21.56925	0.0000
LOG_3Y	1.162377	0.009785	118.7908	0.0000
DUMMY	-0.090199	0.013503	-6.679702	0.0000

R-squared	0.971910	Mean dependent var	1.676341
Adjusted R-squared	0.971812	S.D. dependent var	0.522560
S.E. of regression	0.087734	Akaike info criterion	-2.023797
Sum squared resid	4.395154	Schwarz criterion	-2.001048
Log likelihood	583.8297	F-statistic	9878.373
Durbin-Watson stat	0.109426	Prob(F-statistic)	0.000000

Table 7: Résultats de la régression $\log\_1y_t = \beta_0 + \beta_1 \log\_3y_t + \beta_2 dummy_t + \varepsilon_t$.

Augmented Dickey-Fuller Unit Root Test on ESTIM_RESIDS

Null Hypothesis: ESTIM_RESIDS has a unit root
Exogenous: Constant
Lag Length: 0 (Fixed)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.092138	0.0011
Test critical values:	1% level	-3.441533	
	5% level	-2.866365	
	10% level	-2.569399	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(ESTIM_RESIDS)
Method: Least Squares
Date: 12/08/12 Time: 11:10
Sample (adjusted): 1953M05 2001M01
Included observations: 573 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ESTIM_RESIDS(-1)	-0.055922	0.013666	-4.092138	0.0000
C	-7.11E-05	0.001195	-0.059517	0.9526

R-squared	0.028491	Mean dependent var	-5.98E-05
Adjusted R-squared	0.026790	S.D. dependent var	0.028997
S.E. of regression	0.028606	Akaike info criterion	-4.266938
Sum squared resid	0.467241	Schwarz criterion	-4.251751
Log likelihood	1224.478	F-statistic	16.74560
Durbin-Watson stat	1.771323	Prob(F-statistic)	0.000049

Table 8: Test de cointégration.

Vector Autoregression Estimates

Vector Autoregression Estimates
Date: 12/07/12 Time: 17:06
Sample (adjusted): 1953M07 2001M01
Included observations: 571 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	DLOG_1Y	DLOG_3Y
DLOG_1Y(-1)	0.251306	0.140176
	(0.09141)	(0.07100)
	[2.74927]	[1.97429]
DLOG_1Y(-2)	-0.046402	0.038268
	(0.08973)	(0.06970)
	[-0.51714]	[0.54907]
DLOG_3Y(-1)	0.331380	0.303343
	(0.11635)	(0.09038)
	[2.84804]	[3.35643]
DLOG_3Y(-2)	-0.093708	-0.231284
	(0.11690)	(0.09080)
	[-0.80161]	[-2.54716]
C	0.000654	0.000696
	(0.00254)	(0.00198)
	[0.25733]	[0.35214]
R-squared	0.206883	0.187498
Adj. R-squared	0.201278	0.181756
Sum sq. resids	2.088340	1.259955
S.E. equation	0.060742	0.047181
F-statistic	36.90997	32.65347
Log likelihood	791.7322	935.9935
Akaike AIC	-2.755629	-3.260923
Schwarz SC	-2.717561	-3.222855
Mean dependent	0.001181	0.000971
S.D. dependent	0.067966	0.052159

Determinant resid covariance (dof adj.)	1.70E-06
Determinant resid covariance	1.67E-06
Log likelihood	2177.005
Akaike information criterion	-7.590212
Schwarz criterion	-7.514075

Table 9: Estimation du VAR.

VAR Residual Serial Correlation LM T
H0: no serial correlation at lag order h
Date: 12/07/12 Time: 17:20
Sample: 1953M04 2001M01
Included observations: 571

Lags	LM-Stat	Prob
1	8.319155	0.0806
2	7.541549	0.1099
3	5.123223	0.2749
4	3.536108	0.4724
5	8.320903	0.0805
6	8.085848	0.0885

Probs from chi-square with 4 df.

Table 10: Test de l'estimation du VAR.

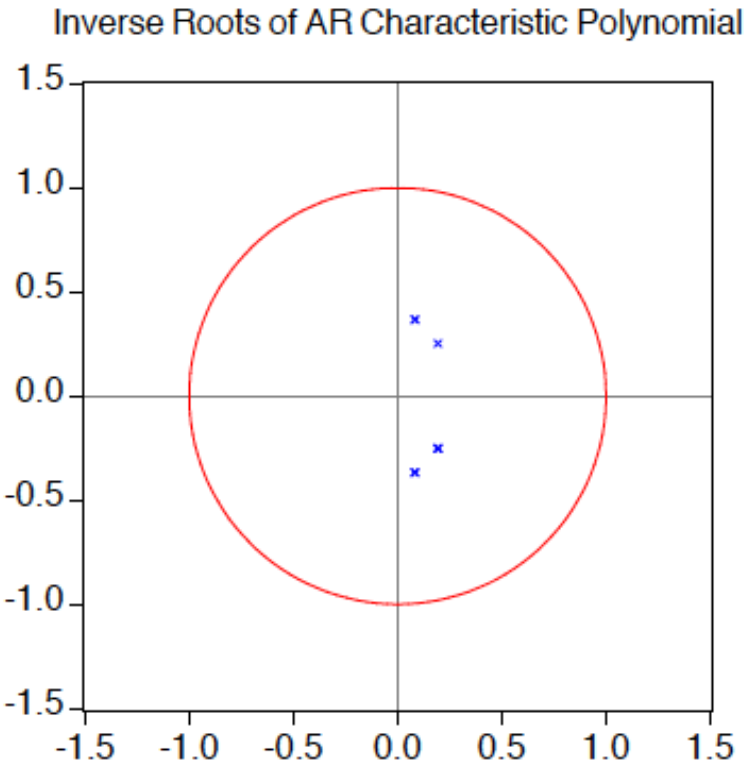

Figure 4: Test de l'estimation du VAR.

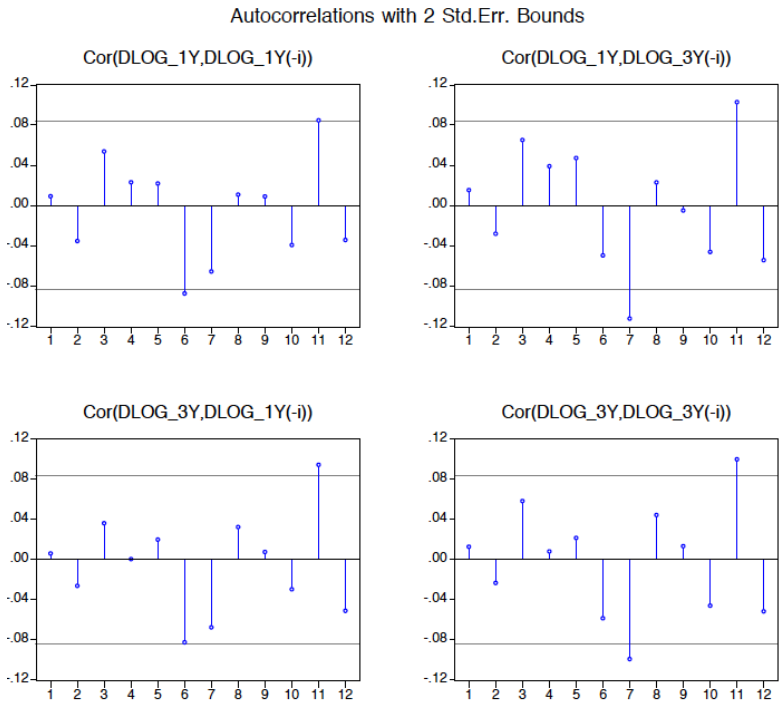

Table 11: Test de l'estimation du VAR.

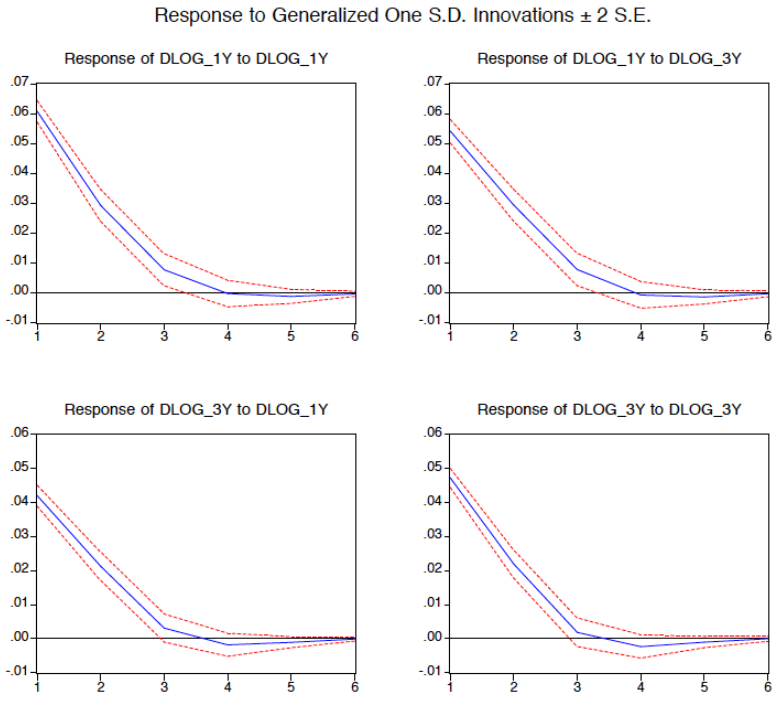

Table 12: Fonctions de réponses impulsionnelles.