Séries Temporelles (4386)

Master I Economie-Gestion

Mention Ingénierie Economique et Statistique et Mention Monnaie-Finance-Banque

Professeur Georges Bresson

1. **Exercice 1** (5.5 pts.)

   Soit le processus $ARMA(1,1)$

   $$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t, \ t = 1, ..., T$$

   avec $X_0 = 0$, $\phi_1 = 0.3$, $\phi_2 = -0.7$. Sachant que $\varepsilon_t \sim N(0, \sigma_\epsilon^2)$ est un bruit blanc gaussien d'espérance nulle et de variance $\sigma_\epsilon^2 = 4$,

   (a) (1 pt.) Calculez les racines λ_1 et λ_2 de l'équation. Le processus est-il stationnaire?

   (b) (0.5 pt.) Déterminez la période du cycle.

   (c) (2 pts.) Sachant que le multiplicateur dynamique à l'horizon τ est défini par

   $$\frac{\partial X_{t+\tau}}{\partial \varepsilon_t} = c_1 \lambda_1^\tau + c_2 \lambda_2^\tau \text{ avec } c_j = \frac{\lambda_j}{\prod_{k=1, \ (k \neq j)}^2 (\lambda_j - \lambda_k)}, j = 1, 2$$

   Donnez l'expression numérique du multiplicateur dynamique cumulé à l'horizon $\tau = 4$.

   (d) (2 pts.) Sachant que $E[X_t] = 0$, calculez les coefficients d'autocorrélation ρ_τ pour $\tau = 1$ et $\tau = 2$ ainsi que la variance σ_X^2 de X_t.

2. **Exercice 2** (14.5 pts.)

   On considère les cours de clôture des actions des GAFA (Google, Apple, Facebook et Amazon) au NYSE (Nasdaq) sur la période du 18 juin 2012 au 31 octobre 2017, soit 1337 observations journalières (voir figure 1). Les séries sont notées close_amazon (ou amazon), close_google (ou google), close_apple (ou apple) et close_facebook (ou facebook) selon les tableaux et figures de résultats.

   (a) (3 pts.) On teste la présence de racine unitaire sur le cours de clôture de l'action Amazon. Commentez les tables 1 à 3. Que concluez-vous? On teste la présence de racine unitaire sur les rendements composés des actions des GAFA (table 4). Les rendements composés sont notés r_{amazon}, r_{google}, r_{apple} et $r_{facebook}$. Que concluez-vous?

   (b) (2 pts.) On estime un modèle ARCH non linéaire sur les rendements composés de l'action Amazon. Commentez les tables 5 et 6.

   (c) (1.5 pt.) A l'aide des valeurs estimées des paramètres, exprimez la volatilité σ_t^2 (et non le log de la volatilité) sous forme d'un modèle GARCH à seuil.

   (d) (2 pts.) Pour un choc de ± 2 écarts-types au temps $(t-1)$ et de ± 1 écart-type au temps $(t-2)$, (i.e., $\varepsilon_{t-1} = 2$ ou -2 et $\varepsilon_{t-2} = 1$ ou -1), déterminez l'impact de ces chocs sur la volatilité. Que concluez-vous?

   (e) (3 pts.) On regresse par les MCO le logarithme du cours de clôture de l'action d'Amazon sur ceux de Google, Apple et Facebook. Commentez la table 7. On effectue un test de racine unitaire sur les résidus d'estimation (table 8). Interprétez les tests de cointégration (racine unitaire et test CRDW). Que concluez-vous?

(f) (3 pts.) On estime un VECM sur les rendements composés des actions des GAFA. Interprétez les résultats des tables 9 et 10. Ce VECM est-il cointégré?

**Aucun document autorisé.
Calculatrices et tables statistiques autorisées.**

Table des valeurs critiques du test de Dickey-Fuller pour $\rho = 1$

T	1%	5%	10%
modèle (1)			
100	-2.60	-1.95	-1.61
250	-2.58	-1.95	-1.62
500	-2.58	-1.95	-1.62
∞	-2.58	-1.95	-1.62
modèle (2)			
100	-3.51	-2.89	-2.58
250	-3.46	-2.88	-2.57
500	-3.44	-2.87	-2.57
∞	-3.43	-2.86	-2.57
modèle (3)			
100	-4.04	-3.45	-3.15
250	-3.99	-3.43	-3.13
500	-3.98	-3.42	-3.13
∞	-3.96	-3.41	-3.12

Table des valeurs critiques de la constante et de la tendance, tests de Dickey-Fuller

	Modèle (2)			Modèle (3)					
	Constante			Constante			Tendance		
T	1%	5%	10%	1%	5%	10%	1%	5%	10%
100	3.22	2.54	2.17	3.78	3.11	2.73	3.53	2.79	2.38
250	3.19	2.53	2.16	3.74	3.09	2.73	3.49	2.79	2.38
500	3.18	2.52	2.16	3.72	3.08	2.72	3.48	2.78	2.38
∞	3.18	2.52	2.16	3.71	3.08	2.72	3.46	2.78	2.38

Table des valeurs critiques des tests DF de cointégration avec constante

variables (y compris Y_t)	1%	5%	10%
2	-3.90	-3.34	-3.04
3	-4.29	-3.74	-3.45
4	-4.64	-4.10	-3.81
5	-4.96	-4.42	-4.13

Table des valeurs critiques du test CRDW à 5%

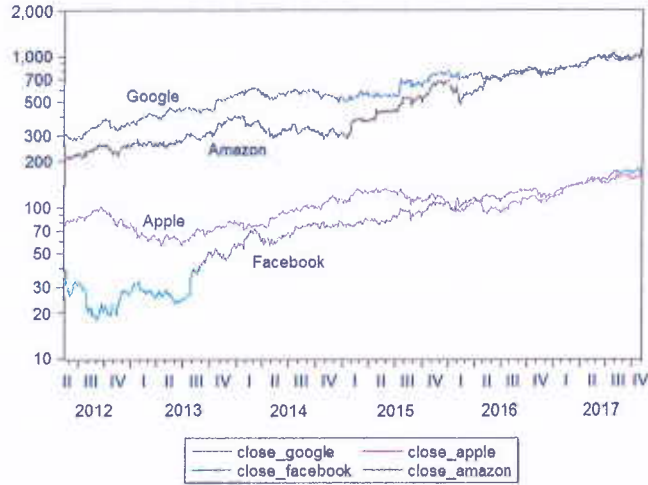| variables (y compris Y_t)	$T = 50$	$T = 100$	$T = 200$
2	0.72	0.38	0.20
3	0.89	0.48	0.25
4	1.05	0.58	0.30
5	1.19	0.68	0.35

Figure 1: Cours de clôture des actions des GAFA au Nasdaq.

Null Hypothesis: CLOSE_AMAZON has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.757589	0.9988
Test critical values:	1% level	-2.566647	
	5% level	-1.941054	
	10% level	-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CLOSE_AMAZON)
Method: Least Squares
Date: 11/24/17   Time: 16:14
Sample: 5/18/2012 10/31/2017
Included observations: 1373

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CLOSE_AMAZON(-1)	0.001233	0.000447	2.757589	0.0059
R-squared	0.000665	Mean dependent var		0.645972
Adjusted R-squared	0.000665	S.D. dependent var		9.256081
S.E. of regression	9.253003	Akaike info criterion		7.288502
Sum squared resid	117468.0	Schwarz criterion		7.292307
Log likelihood	-5002.556	Hannan-Quinn criter.		7.289925
Durbin-Watson stat	1.952333			

Table 1: Premier test de racine unitaire

Null Hypothesis: CLOSE_AMAZON has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.989590	0.9966
Test critical values:	1% level	-3.434903	
	5% level	-2.863438	
	10% level	-2.567830	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CLOSE_AMAZON)
Method: Least Squares
Date: 11/24/17   Time: 16:13
Sample: 5/18/2012 10/31/2017
Included observations: 1373

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CLOSE_AMAZON(-1)	0.000999	0.001009	0.989590	0.3225
C	0.146138	0.563489	0.259344	0.7954

R-squared	0.000714	Mean dependent var	0.645972
Adjusted R-squared	-0.000015	S.D. dependent var	9.256081
S.E. of regression	9.256150	Akaike info criterion	7.289909
Sum squared resid	117462.2	Schwarz criterion	7.297520
Log likelihood	-5002.523	Hannan-Quinn criter.	7.292757
F-statistic	0.979289	Durbin-Watson stat	1.951971
Prob(F-statistic)	0.322549		

Table 2: Deuxième test de racine unitaire

Null Hypothesis: CLOSE_AMAZON has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.527931	0.8198
Test critical values:	1% level	-3.964743	
	5% level	-3.413087	
	10% level	-3.128551	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CLOSE_AMAZON)
Method: Least Squares
Date: 11/24/17   Time: 16:12
Sample: 5/18/2012 10/31/2017
Included observations: 1373

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CLOSE_AMAZON(-1)	-0.004404	0.002882	-1.527931	0.1268
C	0.379546	0.574835	0.660269	0.5092
@TREND("5/18/2012")	0.003602	0.001800	2.000732	0.0456

R-squared	0.003625	Mean dependent var	0.645972
Adjusted R-squared	0.002170	S.D. dependent var	9.256081
S.E. of regression	9.246030	Akaike info criterion	7.288448
Sum squared resid	117120.0	Schwarz criterion	7.299864
Log likelihood	-5000.520	Hannan-Quinn criter.	7.292720
F-statistic	2.492180	Durbin-Watson stat	1.947127
Prob(F-statistic)	0.083104		

Table 3: Troisième test de racine unitaire.

Null Hypothesis: R_AMAZON has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-36.57585	0.0000
Test critical values:	1% level	-2.566650	
	5% level	-1.941055	
	10% level	-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R_AMAZON)
Method: Least Squares
Date: 11/25/17   Time: 19:20
Sample (adjusted): 5/22/2012 10/31/2017
Included observations: 1371 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R_AMAZON(-1)	-0.987707	0.027004	-36.57585	0.0000
R-squared	0.494053	Mean dependent var		-1.81E-05
Adjusted R-squared	0.494053	S.D. dependent var		0.025651
S.E. of regression	0.018245	Akaike info criterion		-5.169085
Sum squared resid	0.456063	Schwarz criterion		-5.165275
Log likelihood	3544.408	Hannan-Quinn criter.		-5.167659
Durbin-Watson stat	1.997763			

Null Hypothesis: R_GOOGLE has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-35.54100	0.0000
Test critical values:	1% level	-2.566650	
	5% level	-1.941055	
	10% level	-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R_GOOGLE)
Method: Least Squares
Date: 11/25/17   Time: 19:21
Sample (adjusted): 5/22/2012 10/31/2017
Included observations: 1371 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R_GOOGLE(-1)	-0.958447	0.026967	-35.54100	0.0000
R-squared	0.479713	Mean dependent var		-1.65E-05
Adjusted R-squared	0.479713	S.D. dependent var		0.019083
S.E. of regression	0.013765	Akaike info criterion		-5.732714
Sum squared resid	0.259563	Schwarz criterion		-5.728904
Log likelihood	3930.776	Hannan-Quinn criter.		-5.731289
Durbin-Watson stat	1.994294			

Null Hypothesis: R_APPLE has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-36.12864	0.0000
Test critical values:	1% level	-2.566650	
	5% level	-1.941055	
	10% level	-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R_APPLE)
Method: Least Squares
Date: 11/25/17   Time: 19:21
Sample (adjusted): 5/22/2012 10/31/2017
Included observations: 1371 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R_APPLE(-1)	-0.971265	0.026884	-36.12864	0.0000
R-squared	0.487903	Mean dependent var		-3.12E-05
Adjusted R-squared	0.487903	S.D. dependent var		0.021695
S.E. of regression	0.015525	Akaike info criterion		-5.492017
Sum squared resid	0.330200	Schwarz criterion		-5.488207
Log likelihood	3765.778	Hannan-Quinn criter.		-5.490591
Durbin-Watson stat	1.994654			

Null Hypothesis: R_FACEBOOK has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-36.48934	0.0000
Test critical values:	1% level	-2.566650	
	5% level	-1.941055	
	10% level	-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R_FACEBOOK)
Method: Least Squares
Date: 11/25/17   Time: 19:22
Sample (adjusted): 5/22/2012 10/31/2017
Included observations: 1371 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R_FACEBOOK(-1)	-0.976505	0.026761	-36.48934	0.0000
R-squared	0.492865	Mean dependent var		8.57E-05
Adjusted R-squared	0.492865	S.D. dependent var		0.032347
S.E. of regression	0.023035	Akaike info criterion		-4.702834
Sum squared resid	0.726967	Schwarz criterion		-4.699024
Log likelihood	3224.793	Hannan-Quinn criter.		-4.701408
Durbin-Watson stat	2.016138			

Table 4: Tests de racine unitaire sur les rendements composés des GAFA.

```
Dependent Variable: DLOG(CLOSE_AMAZON)
Method: ML - ARCH
Date: 11/24/17   Time: 16:35
Sample (adjusted): 5/21/2012 10/31/2017
Included observations: 1372 after adjustments
Convergence achieved after 46 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(7) + C(8)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
    C(9)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(10)*RESID(-1)
    /@SQRT(GARCH(-1)) + C(11)*RESID(-2)/@SQRT(GARCH(-2))
```

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000905	0.000269	3.368642	0.0008
DLOG(CLOSE_AMAZON(-1))	0.072204	0.024596	2.935629	0.0033
DLOG(CLOSE_GOOGLE)	0.636177	0.011497	55.33282	0.0000
DLOG(CLOSE_APPLE)	0.038414	0.010815	3.551948	0.0004
DLOG(CLOSE_FACEBOOK)	0.153269	0.007808	19.62911	0.0000
MA(1)	0.031889	0.025502	1.250454	0.2111
Variance Equation				
C(7)	-9.381754	0.045043	-208.2844	0.0000
C(8)	0.558348	0.034178	16.33655	0.0000
C(9)	0.548060	0.042694	12.83708	0.0000
C(10)	0.163389	0.024821	6.582648	0.0000
C(11)	-0.421326	0.022816	-18.46654	0.0000
R-squared	0.305642	Mean dependent var		0.001197
Adjusted R-squared	0.303100	S.D. dependent var		0.018208
S.E. of regression	0.015201	Akaike info criterion		-5.765926
Sum squared resid	0.315623	Schwarz criterion		-5.724043
Log likelihood	3966.425	Hannan-Quinn criter.		-5.750253
Durbin-Watson stat	2.148085			
Inverted MA Roots	-.03			

Table 5: Estimation ARCH non linéaire des rendements composés de l'action Amazon.


```
Forecast: FIITED_AMAZON
Actual: CLOSE_AMAZON
Forecast sample: 5/18/2012 10/31/2017
Adjusted sample: 5/21/2012 10/31/2017
Included observations: 1372
Root Mean Squared Error    7.243435
Mean Absolute Error        4.246995
Mean Abs. Percent Error    0.935145
Theil Inequality Coefficient  0.006474
  Bias Proportion          0.001204
  Variance Proportion      0.000126
  Covariance Proportion    0.998670
```
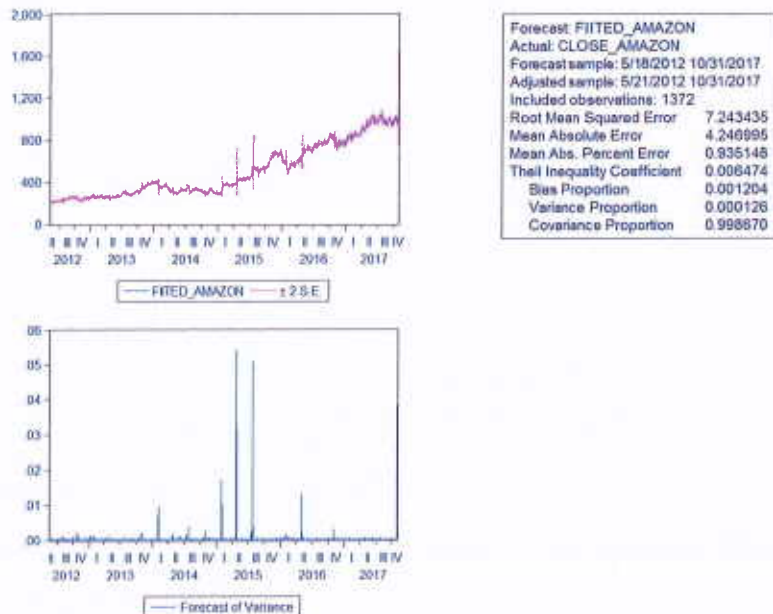
Table 6: Tests de la qualité d'estimation ARCH non linéaire des rendements composés de l'action Amazon.

6

Dependent Variable: LOG(AMAZON)
Method: Least Squares
Date: 11/25/17   Time: 10:58
Sample: 5/18/2012 10/31/2017
Included observations: 1373

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.754850	0.202742	-18.52031	0.0000
LOG(GOOGLE)	1.399633	0.039093	35.80240	0.0000
LOG(APPLE)	0.273213	0.025637	10.65697	0.0000
LOG(FACEBOOK)	-0.077293	0.022953	-3.367460	0.0008
R-squared	0.905672	Mean dependent var		6.099496
Adjusted R-squared	0.905465	S.D. dependent var		0.480190
S.E. of regression	0.147642	Akaike info criterion		-0.985146
Sum squared resid	29.84161	Schwarz criterion		-0.969925
Log likelihood	680.3027	Hannan-Quinn criter.		-0.979450
F-statistic	4381.375	Durbin-Watson stat		0.016285
Prob(F-statistic)	0.000000			

Table 7: Estimation MCO du logarithme du cours de clôture de l'action Amazon.

Null Hypothesis: RESIDS_MCO has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=0)

			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.594746	0.0092
Test critical values:	1% level		-2.566649	
	5% level		-1.941054	
	10% level		-1.616544	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RESIDS_MCO)
Method: Least Squares
Date: 11/25/17   Time: 12:53
Sample (adjusted): 5/21/2012 10/31/2017
Included observations: 1372 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESIDS_MCO(-1)	-0.008921	0.003438	-2.594746	0.0096
R-squared	0.004843	Mean dependent var		-0.000126
Adjusted R-squared	0.004843	S.D. dependent var		0.018827
S.E. of regression	0.018781	Akaike info criterion		-5.111192
Sum squared resid	0.483598	Schwarz criterion		-5.107385
Log likelihood	3507.278	Hannan-Quinn criter.		-5.109767
Durbin-Watson stat	1.975996			

Table 8: Test de cointégration sur les résidus de l'estimation MCO.

Vector Error Correction Estimates
Date: 11/24/17   Time: 18:21
Sample (adjusted): 5/24/2012 10/31/2017
Included observations: 1369 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1			
DLOG(AMAZON(-1))	1.000000			
DLOG(GOOGLE(-1))	0.306800			
	(0.09341)			
	[3.28429]			
DLOG(APPLE(-1))	0.213224			
	(0.07934)			
	[2.68743]			
DLOG(FACEBOOK(-1))	0.412079			
	(0.05426)			
	[7.59384]			
C	-0.002117			

Error Correction:	D(DLOG(AMA	D(DLOG(GOO	D(DLOG(APP	D(DLOG(FAC
CointEq1	-0.668082	-0.383402	-0.267338	-0.585417
	(0.03208)	(0.02576)	(0.03062)	(0.04327)
	[-20.8235]	[-14.8863]	[-8.73127]	[-13.5309]
D(DLOG(AMAZON(-1)))	-0.174615	0.265756	0.183557	0.351593
	(0.03448)	(0.02768)	(0.03291)	(0.04650)
	[-5.06360]	[9.59993]	[5.57753]	[7.56060]
D(DLOG(AMAZON(-2)))	-0.159636	0.132399	0.126126	0.139870
	(0.02912)	(0.02338)	(0.02779)	(0.03927)
	[-5.48196]	[5.66365]	[4.53836]	[3.56178]
D(DLOG(GOOGLE(-1)))	0.165742	-0.579305	0.057370	0.156860
	(0.03710)	(0.02979)	(0.03541)	(0.05004)
	[4.46704]	[-19.4491]	[1.62017]	[3.13499]
D(DLOG(GOOGLE(-2)))	0.219540	-0.282751	-0.018164	0.160366
	(0.03663)	(0.02940)	(0.03496)	(0.04939)
	[5.99371]	[-9.61594]	[-0.51962]	[3.24662]

D(DLOG(APPLE(-1)))	0.073995	0.091632	-0.585941	0.116621
	(0.02853)	(0.02290)	(0.02723)	(0.03848)
	[2.59350]	[4.00069]	[-21.5193]	[3.03108]
D(DLOG(APPLE(-2)))	0.016017	0.072337	-0.296804	0.027326
	(0.02822)	(0.02265)	(0.02693)	(0.03805)
	[0.56762]	[3.19323]	[-11.0211]	[0.71808]
D(DLOG(FACEBOOK(-1)))	0.160843	0.122053	0.064474	-0.494505
	(0.02150)	(0.01726)	(0.02052)	(0.02900)
	[7.47981]	[7.07038]	[3.14169]	[-17.0528]
D(DLOG(FACEBOOK(-2)))	0.132924	0.060911	0.021310	-0.239988
	(0.02009)	(0.01613)	(0.01917)	(0.02709)
	[6.61727]	[3.77728]	[1.11164]	[-8.85938]
C	-2.99E-05	-2.90E-05	-1.99E-05	1.99E-05
	(0.00050)	(0.00040)	(0.00048)	(0.00067)
	[-0.06012]	[-0.07265]	[-0.04185]	[0.02964]
R-squared	0.487238	0.399995	0.342996	0.407962
Adj. R-squared	0.483842	0.396021	0.338645	0.404042
Sum sq. resids	0.461418	0.297359	0.420252	0.839110
S.E. equation	0.018426	0.014792	0.017585	0.024848
F-statistic	143.4834	100.6645	78.83112	104.0514
Log likelihood	3530.247	3830.992	3594.213	3120.890
Akaike AIC	-5.142801	-5.582165	-5.236250	-4.544762
Schwarz SC	-5.104658	-5.544022	-5.198107	-4.506619
Mean dependent	-1.03E-05	-1.05E-05	-7.51E-06	-2.24E-05
S.D. dependent	0.025648	0.019034	0.021624	0.032188

Determinant resid covariance (dof adj.)	9.04E-15
Determinant resid covariance	8.78E-15
Log likelihood	14384.48
Akaike information criterion	-20.95030
Schwarz criterion	-20.78247

Table 9: Estimation VECM sur les rendements composés des actions des GAFA.

Date: 11/25/17   Time: 12:01
Sample (adjusted): 5/24/2012 10/31/2017
Included observations: 1369 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: DLOG(AMAZON) DLOG(GOOGLE) DLOG(APPLE) DLOG(FACEBOOK)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.282075	1595.470	54.07904	1.0000
At most 1 *	0.257834	1141.797	35.19275	0.0000
At most 2 *	0.238499	733.5860	20.26184	0.0001
At most 3 *	0.231559	360.5825	9.164546	0.0001

Trace test indicates 4 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.282075	453.6729	28.58808	0.0001
At most 1 *	0.257834	408.2112	22.29962	0.0000
At most 2 *	0.238499	373.0035	15.89210	0.0001
At most 3 *	0.231559	360.5825	9.164546	0.0001

Max-eigenvalue test indicates 4 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 10: Tests de cointégration multivariée.