

Séries Temporelles

Master I Economie-Gestion

Mention Ingénierie Economique et Statistique et Mention Monnaie-Finance-Banque

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1. **Exercice 1** (3.5 pts.)

Soit le processus $MA(5)$ suivant:

$$X_t = \varepsilon_t - \sum_{j=1}^5 \theta_j \varepsilon_{t-j}, \quad t = 1, \dots, T$$

avec $\theta_1 = \theta_3 = \theta_4 = 0$, $\theta_2 = 0.6$ et $\theta_5 = 0.4$. Sachant que ε_t est un processus bruit blanc d'espérance nulle et de variance $\sigma_\varepsilon^2 = 2$, donnez les expressions de la variance, des covariances et des autocorrélations du processus X_t ainsi que leurs valeurs numériques.

2. **Exercice 2** (9 pts.)

On considère l'indice de volatilité VIX (*Volatility IndeX*) du marché financier américain sur la période 02/janv/1990 - 02/nov/2016. Il est établi quotidiennement par le Chicago Board Options Exchange. Cet indice est calculé en faisant la moyenne des volatilités sur les options d'achat (*call*) et les options de vente (*put*) sur l'indice Standard & Poor's 500 (S&P 500). Cet indice, appelé "indice de la peur", permet de mesurer le niveau de peur des investisseurs qui est implicitement contenu dans les prix des options.

- Commentez la figure 1 et la table 1 du VIX (variable `vix_sp500`).
- On teste la présence de racine unitaire. Commentez les tables 2 à 4. Que concluez-vous?
- On estime l'indice VIX à l'aide d'un modèle ARMA(1,1) et on teste la présence d'effets ARCH. Commentez les tables 5 et 6. Que concluez-vous?
- On estime un modèle ARCH non linéaire. Commentez la table 7.
- A l'aide des valeurs estimées des paramètres, exprimez la volatilité σ_t^2 (et non le log de la volatilité) sous forme d'un modèle GARCH à seuil.
- Pour un choc de 2 écarts-types, de 3 écarts-types et de 4 écarts-types, montrez que l'impact d'un choc négatif n'est pas le même celui d'un choc positif de même amplitude.

3. **Exercice 3** (7.5 pts) - On souhaite utiliser un modèle VAR pour le couple $\{\Delta \ln y_t; \Delta \pi_t\}$ où $\Delta \ln y_t (= (1 - B) \ln y_t)$ est le taux de croissance trimestriel du PIB et $\Delta \pi_t (= (1 - B)\pi_t)$, la variation du taux d'inflation. Les données trimestrielles concernent la France et couvrent la période 1980q1 - 2015q4. La variable PIB (y_t) est notée PIB_DEFL et la variable d'inflation (π_t) est notée INFLATION. Pour tenir compte de l'impact de la crise de 2008, on introduit une dummy valant 1 pour 2008q4 - 2009q4 et 0 sinon.

- Les séries $\Delta \ln y_t$ et $\Delta \pi_t$ sont $I(0)$. Commentez la table 8 du test de causalité "à la Granger". Que concluez-vous?
- On estime un processus VAR non cointégré. Commentez les tables 9 et 10 ainsi que les figures 2 et 3.

- (c) A partir des valeurs de la table 10, donnez les valeurs numériques des fonctions de réponse impulsionnelles cumulées pour $\tau = 1, 2, 3, 4$ et interprétez les résultats pour $\tau = 4$.

Aucun document autorisé.

Calculatrices et tables statistiques autorisées.

Table des valeurs critiques du test de Dickey-Fuller pour $\rho = 1$

T	1%	5%	10%
modèle (1)			
100	-2.60	-1.95	-1.61
250	-2.58	-1.95	-1.62
500	-2.58	-1.95	-1.62
∞	-2.58	-1.95	-1.62
modèle (2)			
100	-3.51	-2.89	-2.58
250	-3.46	-2.88	-2.57
500	-3.44	-2.87	-2.57
∞	-3.43	-2.86	-2.57
modèle (3)			
100	-4.04	-3.45	-3.15
250	-3.99	-3.43	-3.13
500	-3.98	-3.42	-3.13
∞	-3.96	-3.41	-3.12

Table des valeurs critiques de la constante et de la tendance, tests de Dickey-Fuller

T	Modèle (2)			Modèle (3)					
	Constante			Constante			Tendance		
	1%	5%	10%	1%	5%	10%	1%	5%	10%
100	3.22	2.54	2.17	3.78	3.11	2.73	3.53	2.79	2.38
250	3.19	2.53	2.16	3.74	3.09	2.73	3.49	2.79	2.38
500	3.18	2.52	2.16	3.72	3.08	2.72	3.48	2.78	2.38
∞	3.18	2.52	2.16	3.71	3.08	2.72	3.46	2.78	2.38

Indice de volatilité (VIX_SP500)
2 janvier 1990 - 2 Novembre 2016

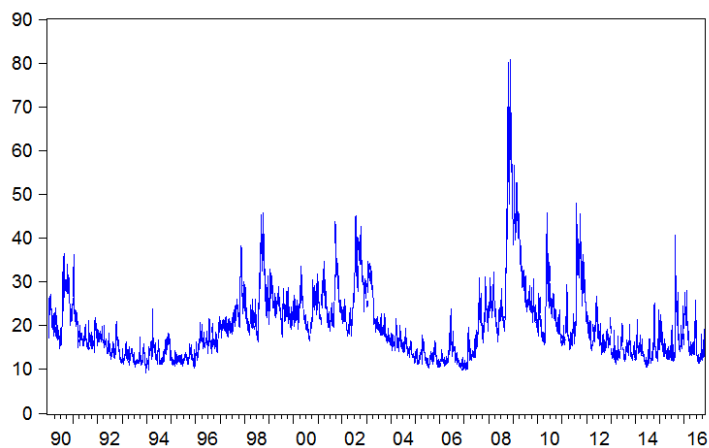


Figure 1: Cours de clôture de l'indice de volatilité.

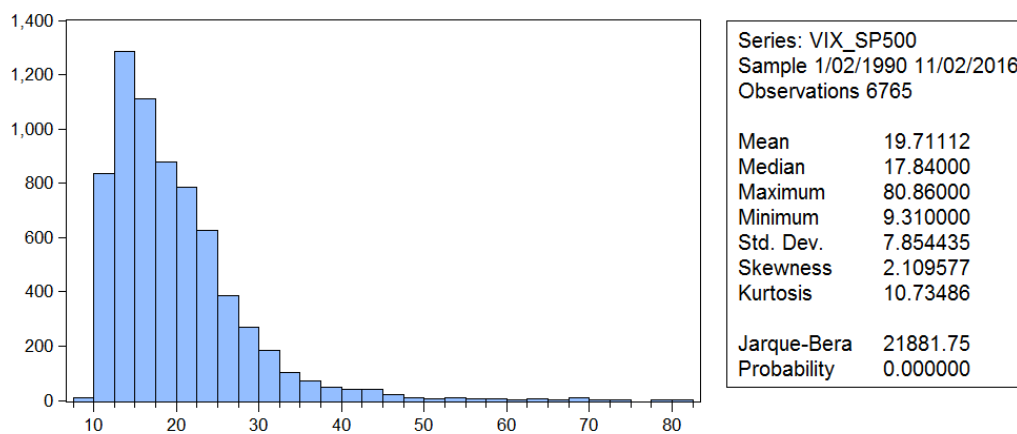


Table 1: Statistiques descriptives du cours de clôture de l'indice de volatilité.

Augmented Dickey-Fuller Unit Root Test on VIX_SP500

Null Hypothesis: VIX_SP500 has a unit root Exogenous: Constant, Linear Trend Lag Length: 5 (Fixed)				
		t-Statistic	Prob.*	
Augmented Dickey-Fuller test statistic:		-5.821553	0.0000	
Test critical values:		1% level	-3.959394	
		5% level	-3.410468	
		10% level	-3.126998	
*MacKinnon (1996) one-sided p-values.				
Augmented Dickey-Fuller Test Equation Dependent Variable: D(VIX_SP500) Method: Least Squares Date: 11/03/16 Time: 19:35 Sample (adjusted): 1/10/1990 11/02/2016 Included observations: 6759 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
VIX_SP500(-1)	-0.013843	0.002378	-5.821553	0.0000
D(VIX_SP500(-1))	-0.105987	0.012227	-8.667989	0.0000
D(VIX_SP500(-2))	-0.096073	0.012253	-7.840771	0.0000
D(VIX_SP500(-3))	-0.058408	0.012277	-4.757375	0.0000
D(VIX_SP500(-4))	-0.071062	0.012225	-5.812933	0.0000
D(VIX_SP500(-5))	-0.019936	0.012168	-1.638406	0.1014
C	0.263654	0.058433	4.512062	0.0000
@TREND("1/02/1990")	2.56E-06	9.41E-06	0.271779	0.7858
R-squared	0.030866	Mean dependent var	-0.000426	
Adjusted R-squared	0.029861	S.D. dependent var	1.530566	
S.E. of regression	1.507541	Akaike info criterion	3.660020	
Sum squared resid	15342.86	Schwarz criterion	3.668090	
Log likelihood	-12361.04	Hannan-Quinn criter.	3.662805	
F-statistic	30.71596	Durbin-Watson stat	2.001362	
Prob(F-statistic)	0.000000			

Table 2: Premier test de racine unitaire du cours de clôture de l'indice de volatilité.

Augmented Dickey-Fuller Unit Root Test on VIX_SP500

Null Hypothesis: VIX_SP500 has a unit root Exogenous: Constant Lag Length: 5 (Fixed)				
		t-Statistic	Prob.*	
Augmented Dickey-Fuller test statistic:		-5.815617	0.0000	
Test critical values:		1% level	-3.431139	
		5% level	-2.861773	
		10% level	-2.566936	
*MacKinnon (1996) one-sided p-values.				
Augmented Dickey-Fuller Test Equation Dependent Variable: D(VIX_SP500) Method: Least Squares Date: 11/03/16 Time: 19:36 Sample (adjusted): 1/10/1990 11/02/2016 Included observations: 6759 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
VIX_SP500(-1)	-0.013814	0.002375	-5.815617	0.0000
D(VIX_SP500(-1))	-0.106006	0.012226	-8.670318	0.0000
D(VIX_SP500(-2))	-0.096092	0.012252	-7.842928	0.0000
D(VIX_SP500(-3))	-0.058426	0.012276	-4.759214	0.0000
D(VIX_SP500(-4))	-0.071079	0.012224	-5.814790	0.0000
D(VIX_SP500(-5))	-0.019952	0.012167	-1.639839	0.1011
C	0.271741	0.050287	5.403817	0.0000
R-squared	0.030855	Mean dependent var	-0.000426	
Adjusted R-squared	0.029994	S.D. dependent var	1.530566	
S.E. of regression	1.507438	Akaike info criterion	3.659735	
Sum squared resid	15343.03	Schwarz criterion	3.666796	
Log likelihood	-12361.07	Hannan-Quinn criter.	3.662172	
F-statistic	35.82789	Durbin-Watson stat	2.001360	
Prob(F-statistic)	0.000000			

Table 3: Deuxième test de racine unitaire du cours de clôture de l'indice de volatilité.

Augmented Dickey-Fuller Unit Root Test on VIX_SP500

Null Hypothesis: VIX SP500 has a unit root Exogenous: None Lag Length: 10 (Fixed)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.010133	0.0426
Test critical values:			1% level	-2.565304
			5% level	-1.940871
			10% level	-1.616868
*MacKinnon (1996) one-sided p-values.				
Augmented Dickey-Fuller Test Equation Dependent Variable: D(VIX SP500) Method: Least Squares Date: 11/03/16 Time: 19:37 Sample (adjusted): 1/17/1990 11/02/2016 Included observations: 8754 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
VIX SP500(-1)	-0.001739	0.000865	-2.010133	0.0445
D(VIX SP500(-1))	-0.118306	0.012169	-9.721986	0.0000
D(VIX SP500(-2))	-0.107156	0.012253	-8.744913	0.0000
D(VIX SP500(-3))	-0.070504	0.012312	-5.726482	0.0000
D(VIX SP500(-4))	-0.084879	0.012314	-6.892696	0.0000
D(VIX SP500(-5))	-0.036418	0.012347	-2.949630	0.0032
D(VIX SP500(-6))	-0.043142	0.012344	-3.494983	0.0005
D(VIX SP500(-7))	-0.068817	0.012310	-5.411426	0.0000
D(VIX SP500(-8))	-0.020812	0.012306	-1.691211	0.0908
D(VIX SP500(-9))	-0.003060	0.012235	-0.250093	0.8025
D(VIX SP500(-10))	0.056237	0.012146	4.630236	0.0000
R-squared	0.036047	Mean dependent var	-0.000720	
Adjusted R-squared	0.034618	S.D. dependent var	1.529469	
S.E. of regression	1.502762	Akaike info criterion	3.654114	
Sum squared resid	15227.67	Schwarz criterion	3.665218	
Log likelihood	-12328.94	Hannan-Quinn criter.	3.657947	
Durbin-Watson stat	1.996838			

Table 4: Troisième test de racine unitaire du cours de clôture de l'indice de volatilité.

Dependent Variable: VIX_SP500 Method: ARMA Maximum Likelihood (OPG - BHHH) Date: 12/05/16 Time: 18:09 Sample: 1/02/1990 11/02/2016 Included observations: 6765 Convergence achieved after 68 iterations Coefficient covariance computed using outer product of gradients				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	19.68437	1.428596	13.77883	0.0000
AR(1)	0.984822	0.001065	924.3116	0.0000
MA(1)	-0.106611	0.005640	-18.90397	0.0000
SIGMASQ	2.298105	0.013331	172.3911	0.0000
R-squared	0.962743	Mean dependent var	19.71112	
Adjusted R-squared	0.962727	S.D. dependent var	7.854435	
S.E. of regression	1.516398	Akaike info criterion	3.671631	
Sum squared resid	15546.68	Schwarz criterion	3.675663	
Log likelihood	-12415.29	Hannan-Quinn criter.	3.673023	
F-statistic	58236.61	Durbin-Watson stat	1.982212	
Prob(F-statistic)	0.000000			
Inverted AR Roots	.98			
Inverted MA Roots	.11			

Table 5: Estimation ARMA du cours de clôture de l'indice de volatilité.

Heteroskedasticity Test: ARCH				
F-statistic	218.8315	Prob. F(10,6744)	0.0000	
Obs*R-squared	1654.898	Prob. Chi-Square(10)	0.0000	
Test Equation: Dependent Variable: RESID^2 Method: Least Squares Date: 12/05/16 Time: 18:10 Sample (adjusted): 1/16/1990 11/02/2016 Included observations: 6755 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.556618	0.116404	4.781788	0.0000
RESID^2(-1)	0.052482	0.012167	4.313440	0.0000
RESID^2(-2)	0.314436	0.012152	25.87425	0.0000
RESID^2(-3)	0.025645	0.012729	2.014740	0.0440
RESID^2(-4)	0.008336	0.012688	0.656994	0.5112
RESID^2(-5)	0.114743	0.012688	9.043094	0.0000
RESID^2(-6)	-0.000765	0.012689	-0.060283	0.9519
RESID^2(-7)	0.087041	0.012688	6.860040	0.0000
RESID^2(-8)	0.046324	0.012728	3.639426	0.0003
RESID^2(-9)	0.067659	0.012152	5.567522	0.0000
RESID^2(-10)	0.040546	0.012163	3.333438	0.0009
R-squared	0.244989	Mean dependent var	2.296077	
Adjusted R-squared	0.243869	S.D. dependent var	10.26905	
S.E. of regression	8.929538	Akaike info criterion	7.218233	
Sum squared resid	537744.0	Schwarz criterion	7.229336	
Log likelihood	-24368.58	Hannan-Quinn criter.	7.222066	
F-statistic	218.8315	Durbin-Watson stat	2.000829	
Prob(F-statistic)	0.000000			

Table 6: Test de la présence d'effets ARCH sur le cours de clôture de l'indice de volatilité.

Dependent Variable: VIX_SP500				
Method: ML - ARCH				
Date: 11/03/16 Time: 19:15				
Sample: 1/02/1990 11/02/2016				
Included observations: 6765				
Convergence achieved after 75 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(6)*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	20.41979	0.981933	20.79550	0.0000
AR(1)	0.986336	0.001070	921.8560	0.0000
MA(1)	-0.076724	0.011299	-6.790454	0.0000
Variance Equation				
C(4)	-0.050023	0.003893	-12.84978	0.0000
C(5)	0.082237	0.005103	16.11413	0.0000
C(6)	0.207743	0.005339	38.91224	0.0000
C(7)	0.961091	0.002020	475.8349	0.0000
R-squared	0.962708	Mean dependent var	19.71112	
Adjusted R-squared	0.962697	S.D. dependent var	7.854435	
S.E. of regression	1.517002	Akaike info criterion	3.102372	
Sum squared resid	15561.37	Schwarz criterion	3.109428	
Log likelihood	-10486.77	Hannan-Quinn criter.	3.104807	
Durbin-Watson stat	2.039704			
Inverted AR Roots	.99			
Inverted MA Roots	.08			

Table 7: Estimation ARCH non linéaire du cours de clôture de l'indice de volatilité.

Pairwise Granger Causality Tests			
Date: 07/20/16 Time: 16:13			
Sample: 1980Q1 2015Q4			
Lags: 3			
Null Hypothesis:	Obs	F-Statistic	Prob.
D(INFLATION) does not Granger Cause DLOG_PIB	99	6.57278	0.0004
DLOG_PIB does not Granger Cause D(INFLATION)		2.32472	0.0800

Table 8: Test de causalité entre $\Delta \ln y_t$ et $\Delta \pi_t$.

Vector Autoregression Estimates

Vector Autoregression Estimates		
Date: 07/21/16 Time: 10:35		
Sample (adjusted): 1991Q2 2015Q4		
Included observations: 99 after adjustments		
Standard errors in () & t-statistics in []		
DLOG(PIB_DED(INFLATION))		
DLOG(PIB_DEFL(-1))	0.503074 (0.14353) [3.50512]	0.144939 (0.09105) [1.59185]
DLOG(PIB_DEFL(-2))	0.139165 (0.14870) [0.93588]	-0.037807 (0.09433) [-0.40078]
DLOG(PIB_DEFL(-3))	-0.300263 (0.13290) [-2.25939]	-0.196813 (0.08431) [-2.33448]
D(INFLATION(-1))	-0.521458 (0.19288) [-2.70354]	-1.078979 (0.12236) [-8.81803]
D(INFLATION(-2))	-0.750458 (0.21854) [-3.43390]	-0.920683 (0.13864) [-6.64074]
D(INFLATION(-3))	-0.568054 (0.13990) [-4.06036]	-0.654919 (0.08875) [-7.37917]
C	0.005369 (0.00141) [3.80103]	0.000701 (0.00090) [0.78194]
DUMMY_2008	-0.010543 (0.00325) [-3.24573]	-0.005645 (0.00208) [-2.73935]
R-squared	0.388067	0.629651
Adj. R-squared	0.340995	0.601163
Sum sq. resids	0.002610	0.001051
S.E. equation	0.005358	0.003398
F-statistic	8.244161	22.10203
Log likelihood	381.4238	426.4780
Akaike AIC	-7.543910	-8.454101
Schwarz SC	-7.334204	-8.244394
Mean dependent	0.007547	-8.38E-05
S.D. dependent	0.006598	0.005380
Determinant resid covariance (dof adj.)		1.78E-10
Determinant resid covariance		1.50E-10
Log likelihood		838.6745
Akaike information criterion		-16.61989
Schwarz criterion		-16.20027

Table 9: Estimation du VAR.

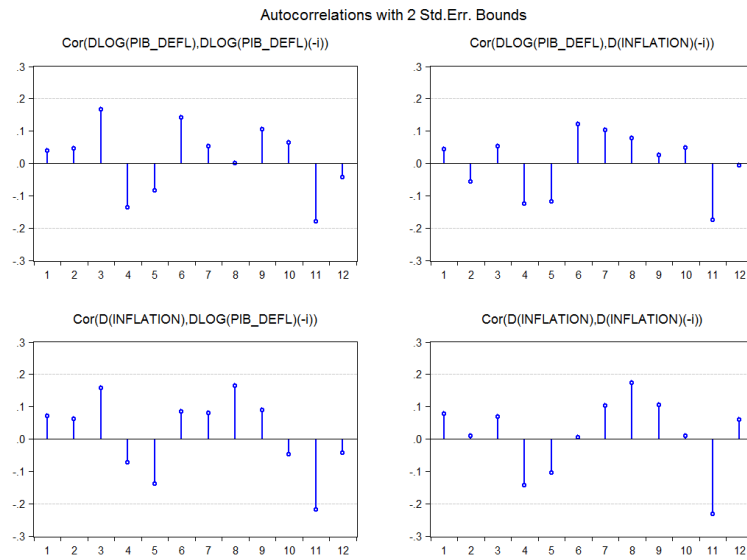


Figure 2: Corrélations croisées des résidus.

Impulse Response to Cholesky (d.f. adjusted) One S.D. Innovations

Response of DLOG(PIB_DEFL): Period DLOG(PIB_DEFL) D(INFLATION)		
1	0.005356	0.000000
2	0.001489	-0.001298
3	0.000655	-0.001121
4	-0.000959	-0.000361
5	0.000681	0.001120
6	-0.000188	-0.000285
7	-5.14E-05	-0.000216
8	-0.000516	-0.000206
9	0.000479	0.000614
10	-5.51E-05	-0.000236
11	3.97E-05	-9.83E-05
12	-0.000289	-0.000144
Response of D(INFLATION): Period DLOG(PIB_DEFL) D(INFLATION)		
1	0.002312	0.002490
2	-0.001718	-0.002687
3	-0.000261	0.000418
4	-0.000666	0.000278
5	0.001627	0.001320
6	-0.000966	-0.001558
7	0.000116	0.000271
8	-0.000436	3.69E-05
9	0.000960	0.000766
10	-0.000612	-0.000898
11	0.000137	0.000223
12	-0.000300	-4.16E-05

Cholesky Ordering: DLOG(PIB_DEFL)

Table 10: Fonctions de réponses impulsionnelles.

Accumulated Response to Cholesky One S.D. Innovations – 2 S.E.

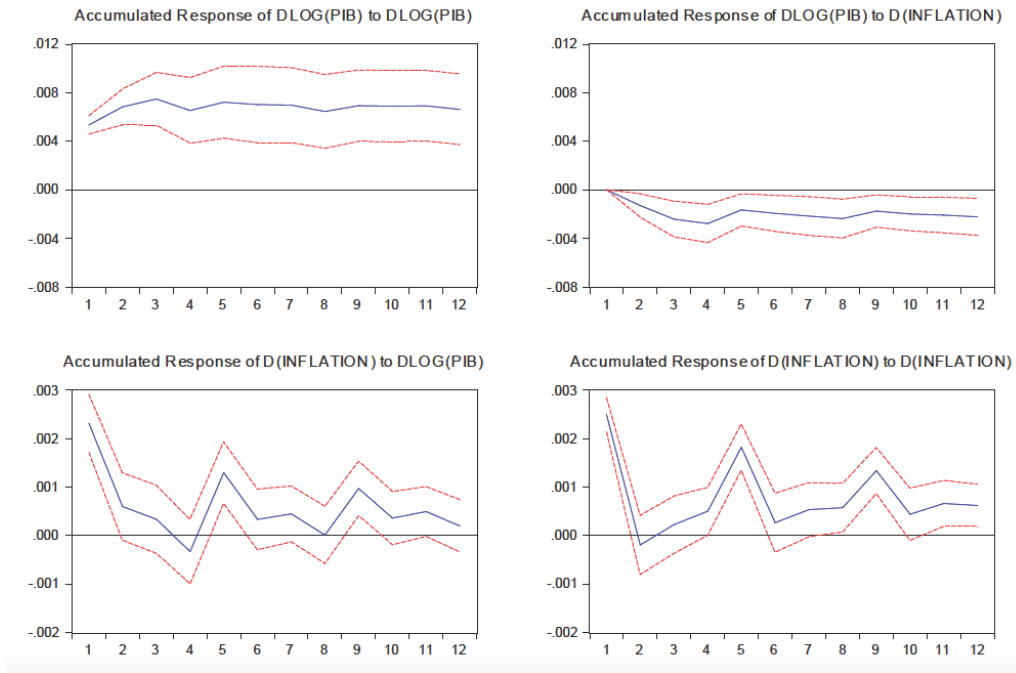


Figure 3: Fonctions de réponses impulsionnelles cumulées.